

Pion/Muon Production using MARS

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From work in collaboration with
C. Johnstone and **MARS** (i.e., Nikolia Mokhov)

Pion/Muon Production:

- Choosing a target...
 - ...Mercury jet
 - ...Graphite rod
 - ...other possibilities?
- The pion decay channel...
 - ...Dominant decay mode ($\sim 100\%$):
$$\pi^+ \rightarrow \mu^+ \nu_\mu$$
$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$
 - ...Lifetime: $\tau = 26$ ns
 - or $c\tau = 7.8$ m
 - ...After a 40 m decay channel:
 - $\sim 99\%$ of pions with $p = 150$ MeV decay
 - $\sim 95\%$ of pions with $p = 250$ MeV decay
 - $\sim 87\%$ of pions with $p = 350$ MeV decay
 - ...Use solenoids to contain the beam
- Matching sections...
 - ...To match from target region into decay channel, use adiabatic magnetic regions ("magnetic horns")
 - ...Use something else to match into the buncher (see C. Johnstone ...sorry Carol!)

The Target Region:

- Using Feasibility Study I target design...
 - ...Graphite rod, 80 cm long, 1.5 cm diameter at a 50 mrad angle w.r.t. central axis (but parallel to incident proton beam)
 - ...1 MW incident proton beam power with 16 GeV protons in pulses at 15 Hz
 - ...Allows for $\sim 5 \times 10^{12}$ muons per pulse if ~ 0.2 muons produced per proton on target
- “Large” capture solenoid...
 - ...Chosen to capture pions of given p_T ,
$$(p_T)_{MAX} = e B_0 \left(\frac{R_0}{2} \right)$$
where B_0 is the strength of the capture solenoid, and R_0 is the radius of the beampipe (one-half the full aperture) at the target ($z = 0$ cm)
 - ...Aperture limited to 60 cm at the end of the decay channel (beginning of buncher)
 - ...Field strength in buncher set at $B = 1.25$ T
 - ...Hence, magnetic flux fixed at the end of decay channel:
$$\Phi \approx B (\pi R^2) = 0.353 \text{ Wb}$$

- The capture solenoid (*continued*)...

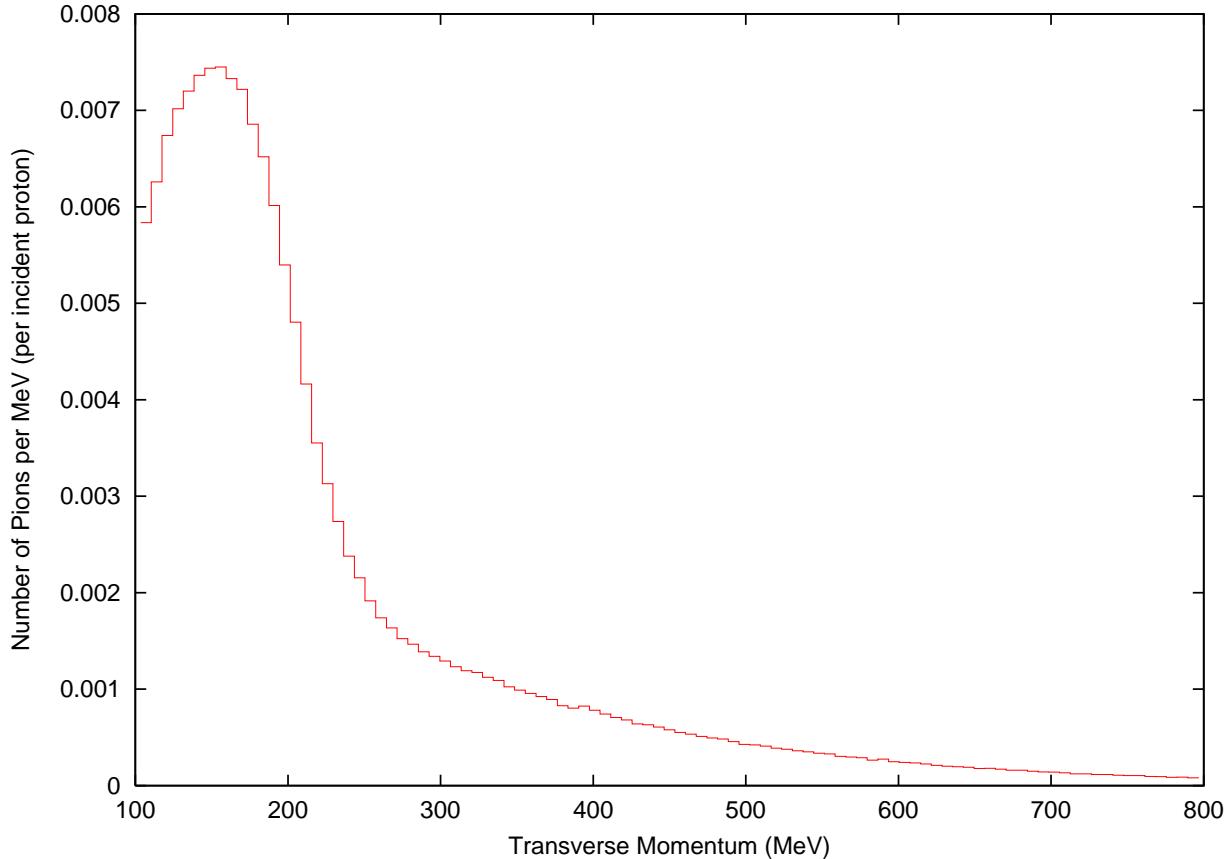
...By matching the sections adiabatically, magnetic flux is conserved throughout the decay channel. So:

$$\Phi_0 \approx B_0 (\pi R_0^2) = \Phi$$

...Hence, increasing the captured $(p_T)_{MAX}$ means decreasing the initial aperture:

$$(p_T)_{MAX} = \frac{e \Phi}{2\pi R_0}$$

...How high should $(p_T)_{MAX}$ be?



How about ~ 225 MeV?

- The capture solenoid (*continued*)...

...With the choice $(p_T)_{MAX} = 225$ MeV,
we find:

$$R_0 = \frac{e \Phi}{2\pi (p_T)_{MAX}} = 7.5 \text{ cm}$$

$$B_0 = \frac{\Phi}{\pi R_0^2} = 20 \text{ T}$$

...Strong! But is it too strong?

Let's assume/hope not...

Matching Sections:

- How to match solenoids into solenoids...

...Use solenoids!

...Simultaneously change field strength and aperture with z , keeping the total flux through the beampipe fixed:

$$B_0 R_0^2 = B(z) R(z)^2$$

...Change must be “slow” in order to be adiabatic, namely:

$$a \ll R_B, B \left(\frac{\partial B}{\partial z} \right)^{-1}$$

where a is the particle's gyroscopic radius of orbit, and R_B is the radius of curvature of the field lines

...Design aperture, $R(z)$, with these constraints and fit $B(z)$ accordingly (using short solenoids) such that:

$$B(z) = B_0 \left(\frac{R_0}{R(z)} \right)^2$$

is the field strength on axis

- Adiabatic matching sections...

...Should hold pions with $P_T < 225$ MeV

...Easily understood in terms of the
adiabatic invariants of motion:

$$Ba^2 \quad \frac{p_T^2}{B}$$

(See Jackson, Sect. 12.6)

...As the particle moves along z , B decreases.

So, p_T decreases and a increases

...Since kinetic energy is conserved, the
longitudinal momentum p_z increases

BOTTOM LINE:

It decreases the divergence of the beam
at the cost of increased beam width

- Designing the “horn” ...

...Determine constraints on aperture of adiabatic region:

$$R(z_1) \equiv R_1 \quad \left(\frac{\partial R}{\partial z} \right) |_{z_1} \equiv \lambda_1$$

$$R(z_2) \equiv R_2 \quad \left(\frac{\partial R}{\partial z} \right) |_{z_2} \equiv \lambda_2$$

...Simple choice for $R(z)$:

$$R(z) \equiv \left(\alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3 \right)^{\frac{1}{k}}$$

Solve for α_i 's as functions of k

...Easy! But how do we choose k ?

Minimize curvature & maximize length!
(i.e., choose $k \approx 1$)

...Example: ($z_1 = 0$ cm, $z_2 = 240$ cm)

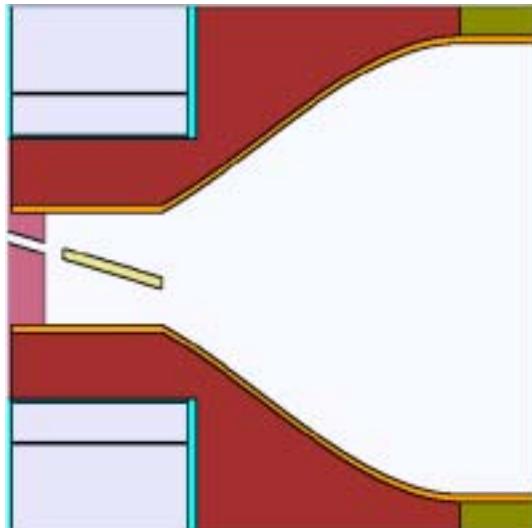
$$R_1 = 7.5 \text{ cm} \quad \lambda_1 = \frac{R_1}{\ell_{tgt}} \sim 0.1$$

$$R_2 = 30 \text{ cm} \quad \lambda_2 = 0$$

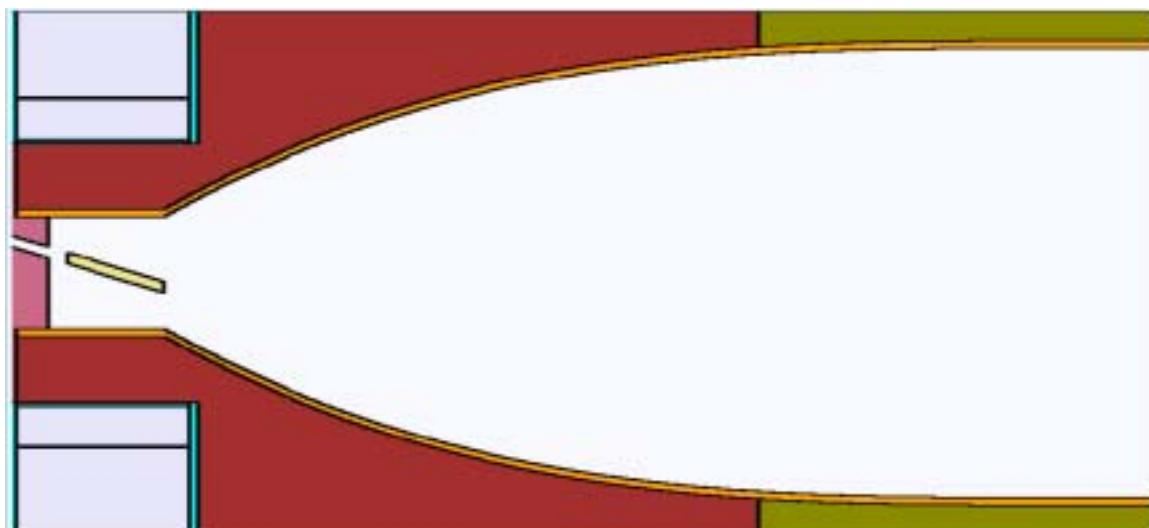
where $\ell_{tgt} = 80$ cm is the length
of the graphite rod target

- Effects of the horn...

...Consider a “short” horn (240 cm):

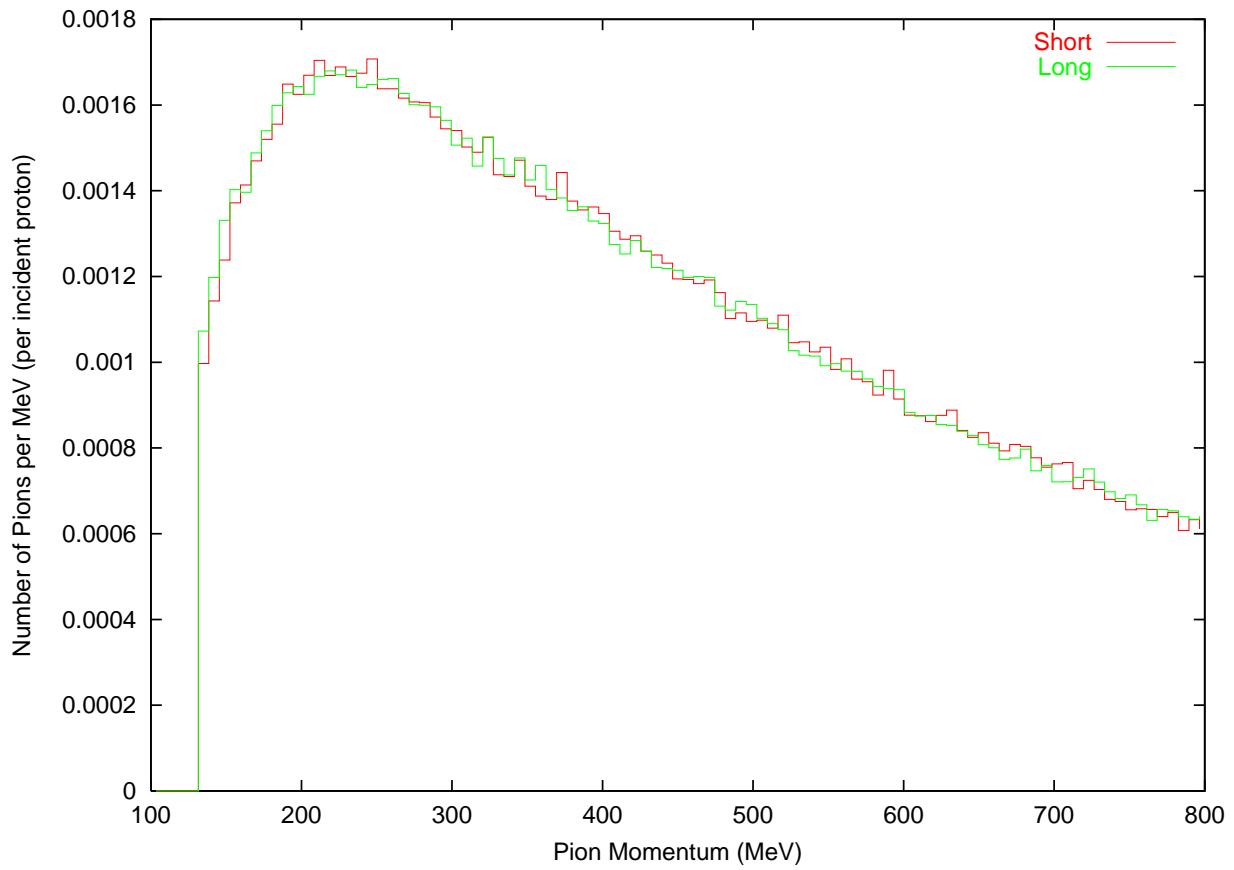


...And a “long” horn (720 cm):



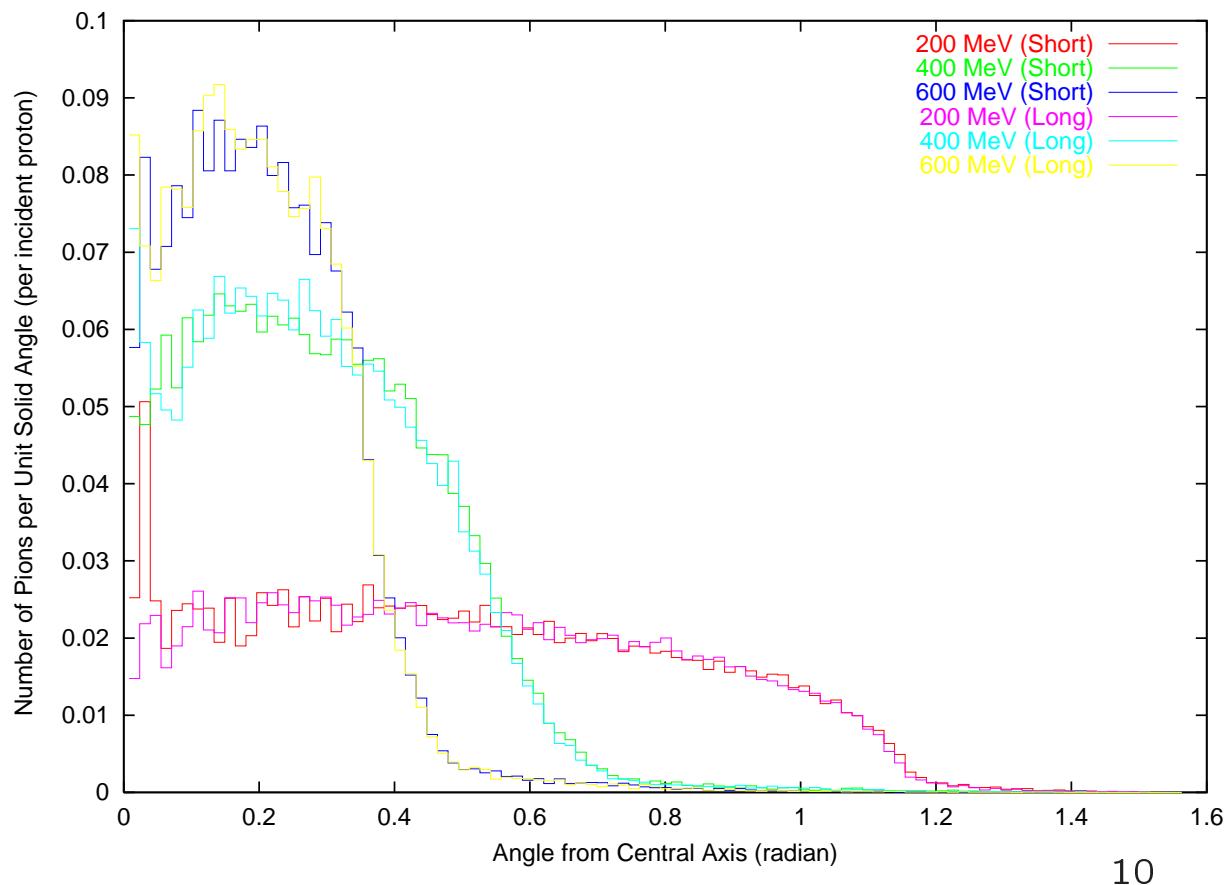
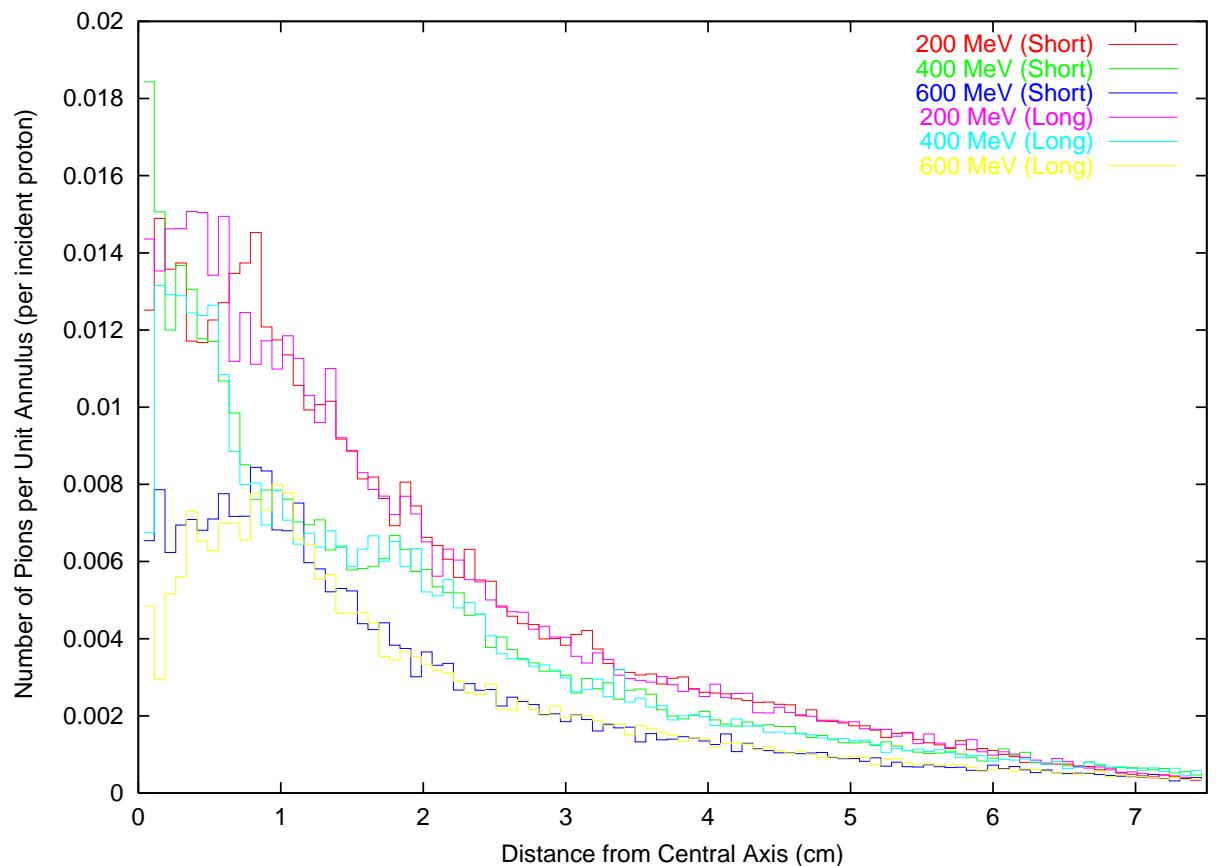
...Consider the number of pions captured

Initial momentum distributions:



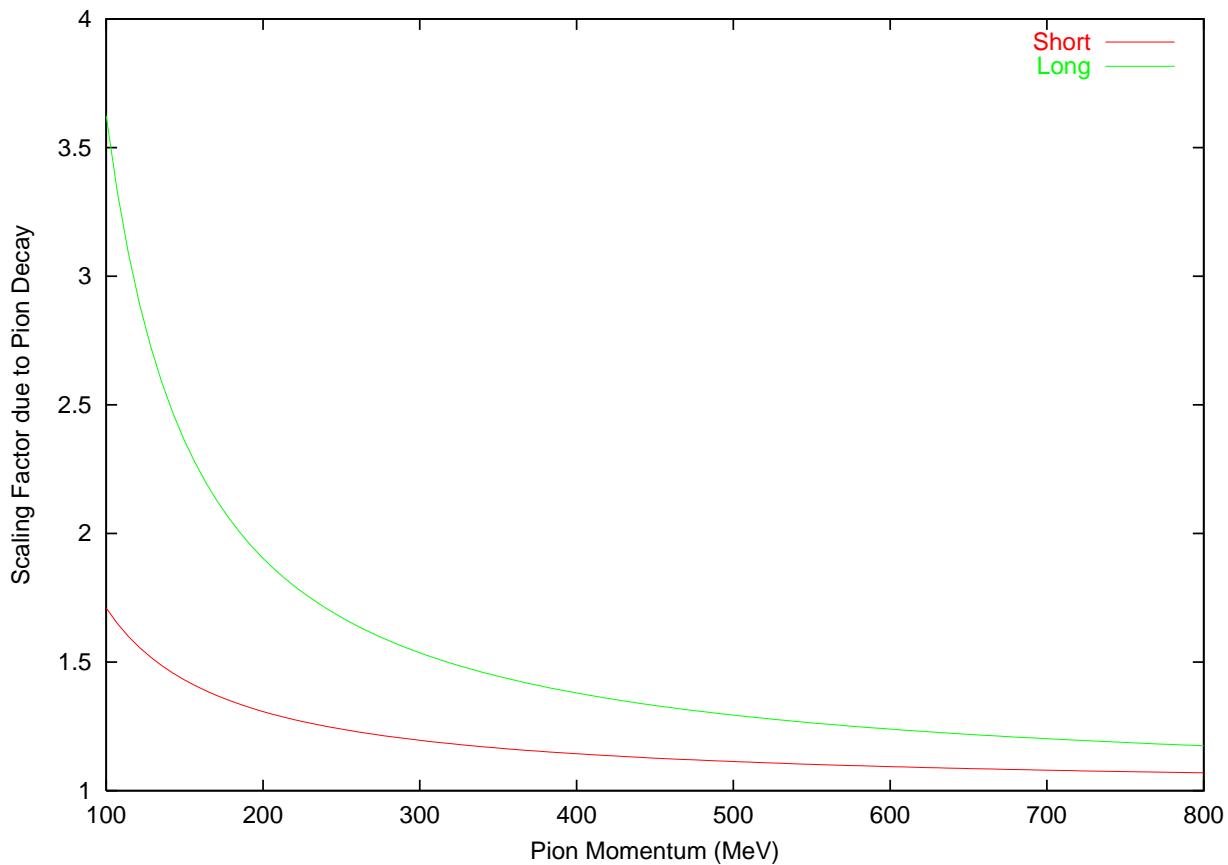
...Similar to p_T distribution

Initial phase-space distributions:



...Distributions at the end of the horns
include losses due to pion decay.

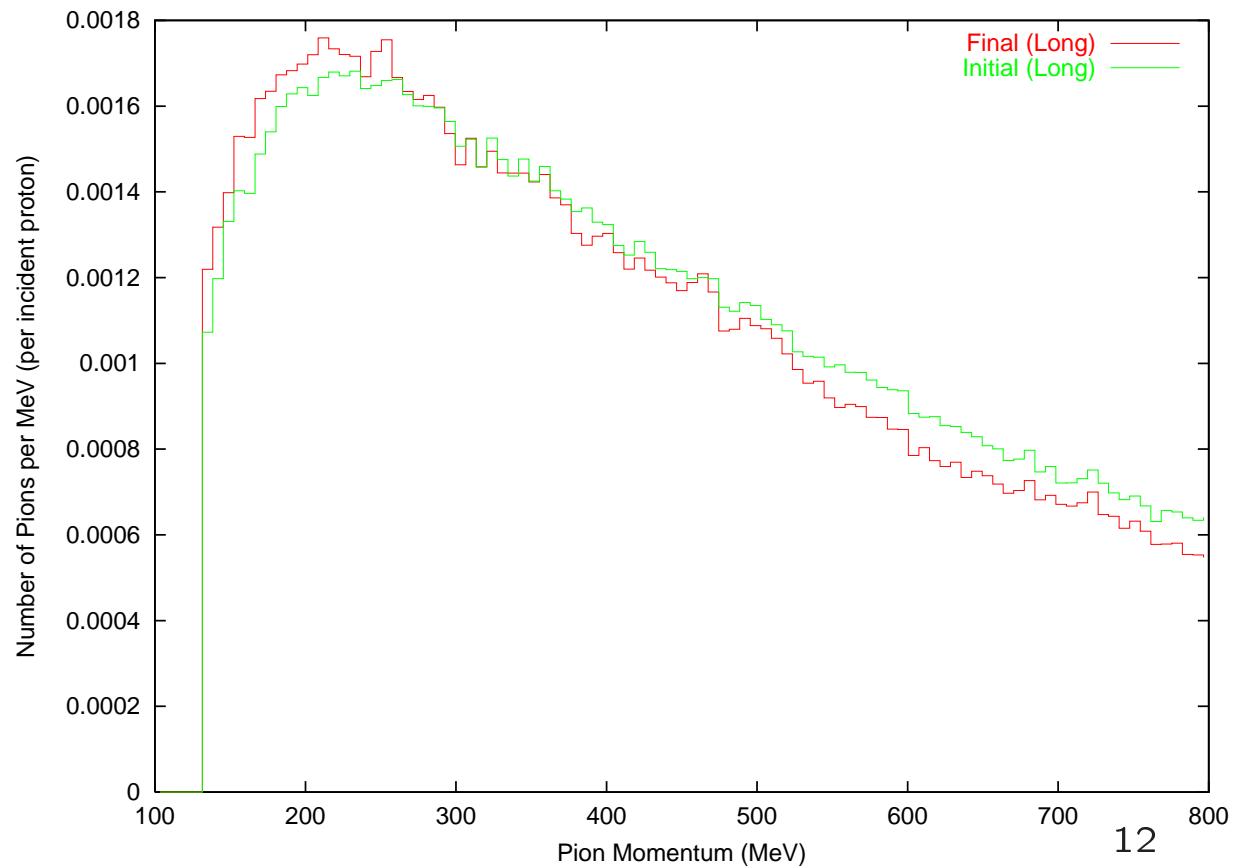
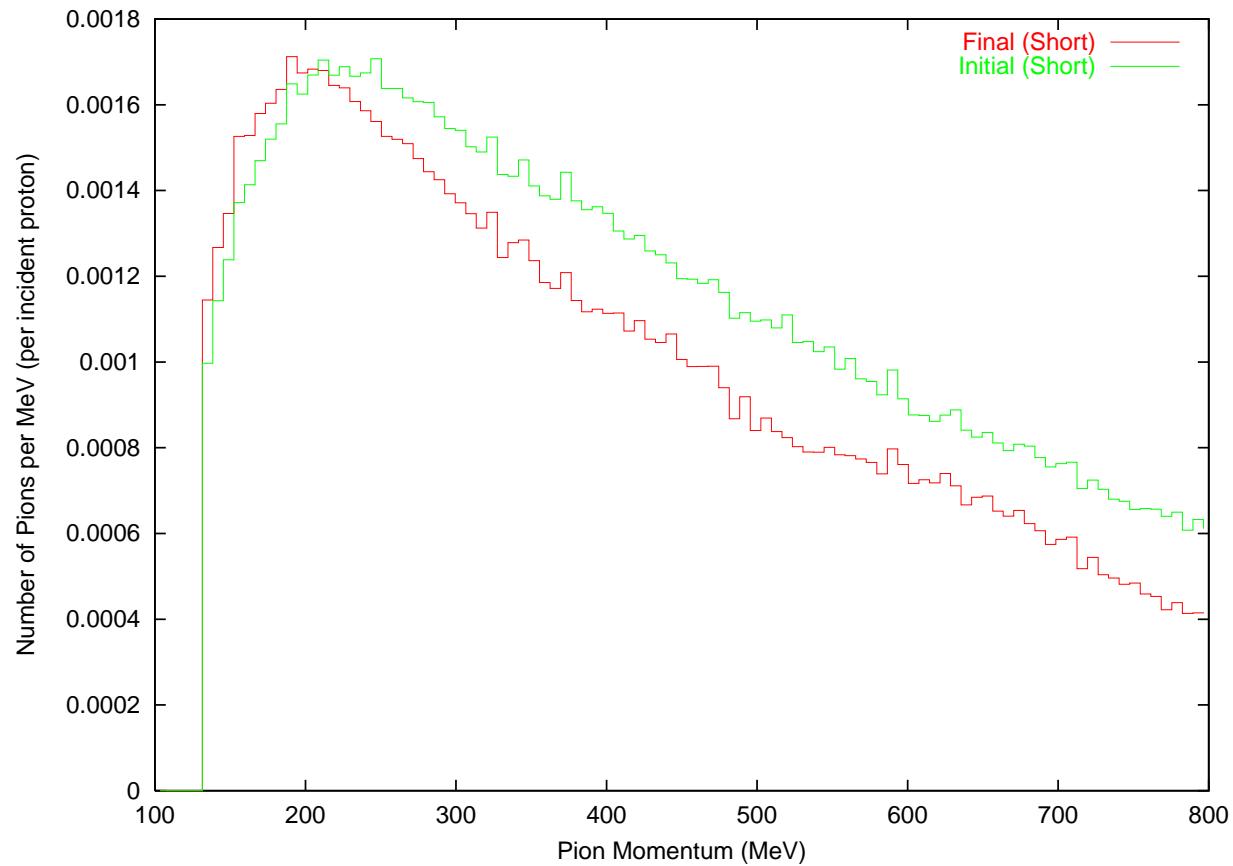
...So, we scale them, according to the pion's
momentum, in order to compare them to their
initial distributions



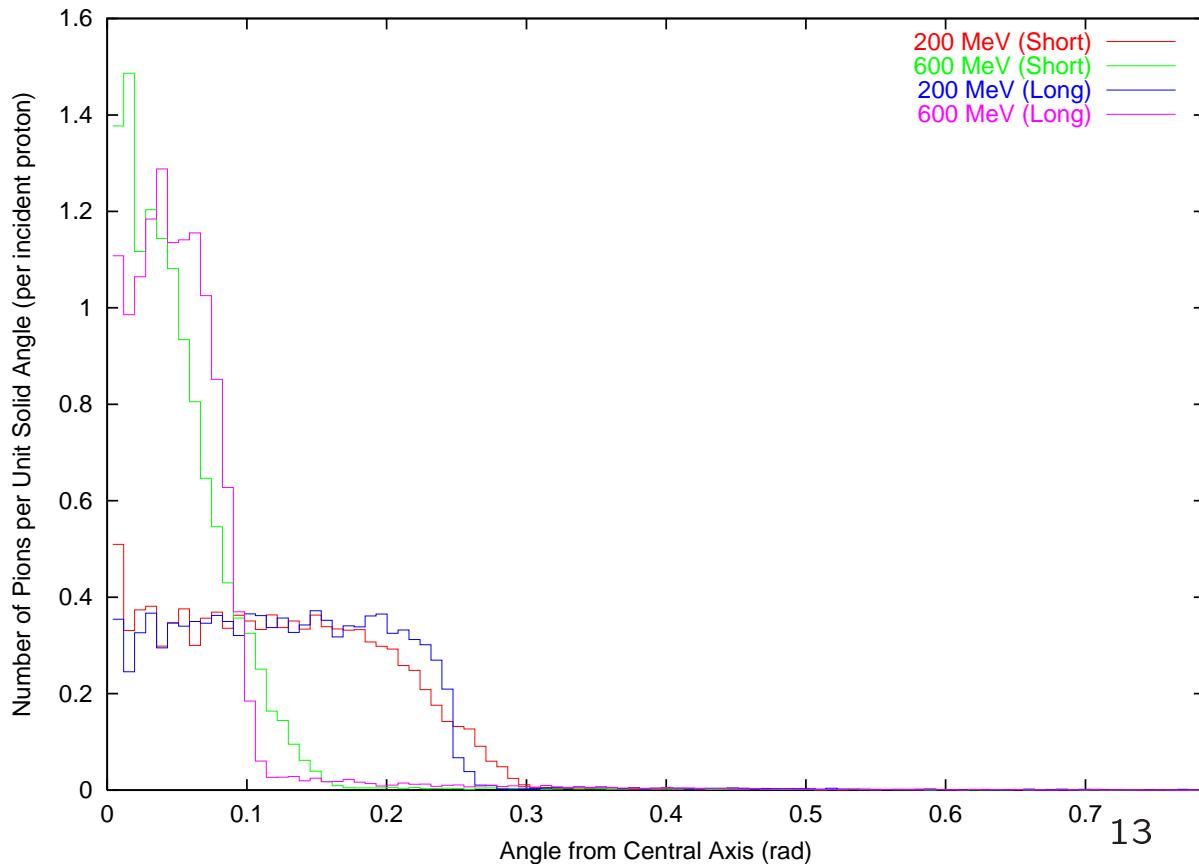
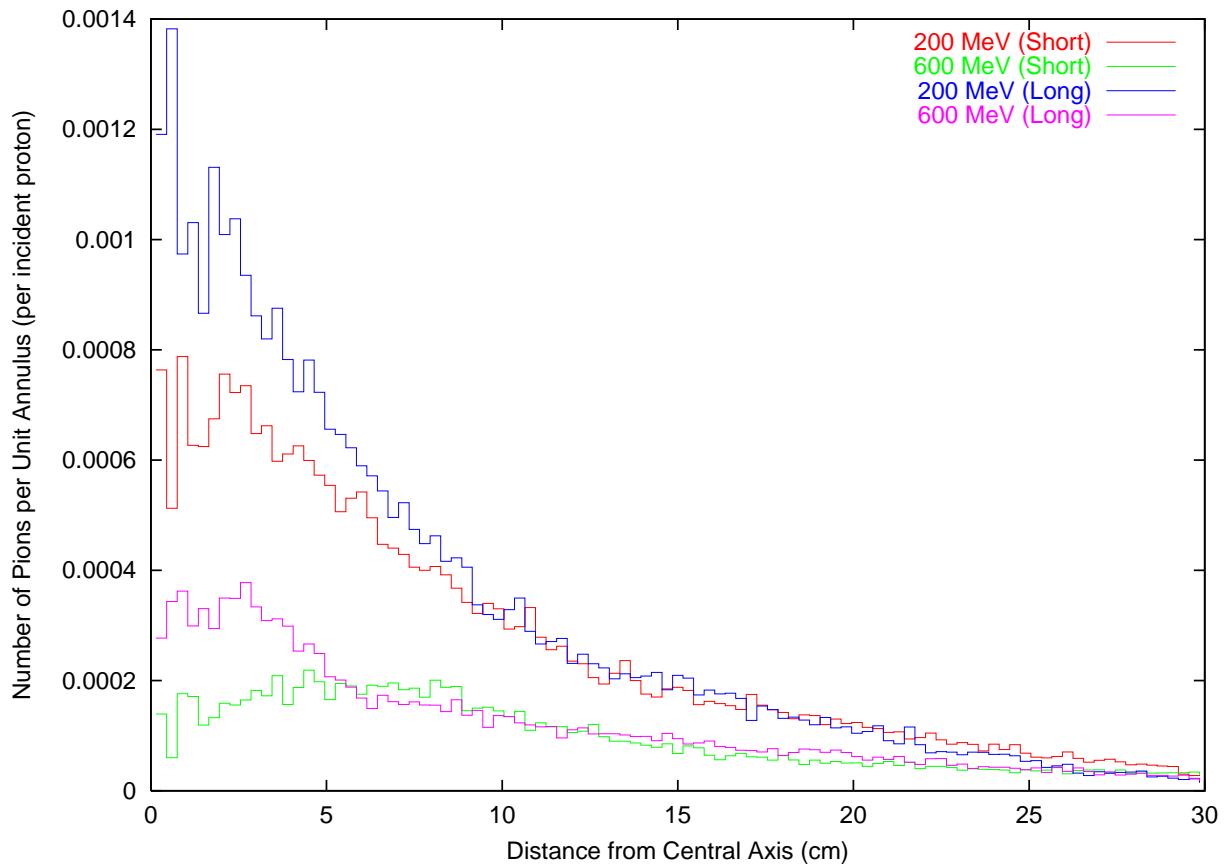
Not accurate for low momenta!

But it allows easier comparison

...Scaled momentum distributions



...Scaled phase space distributions



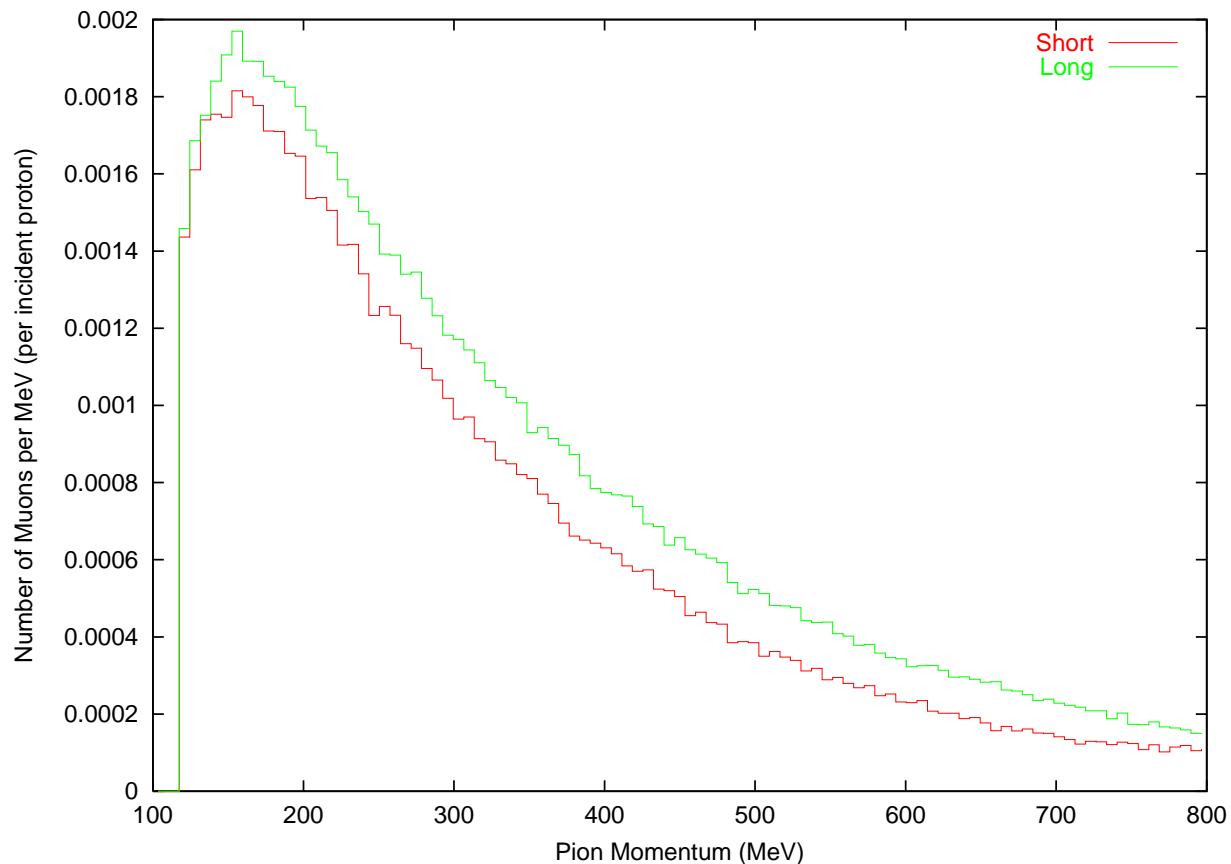
...The final tally: (100 – 800 MeV)

SHORT: 0.241(+), 0.219(-) μ/P

LONG: 0.289(+), 0.258(-) μ/P

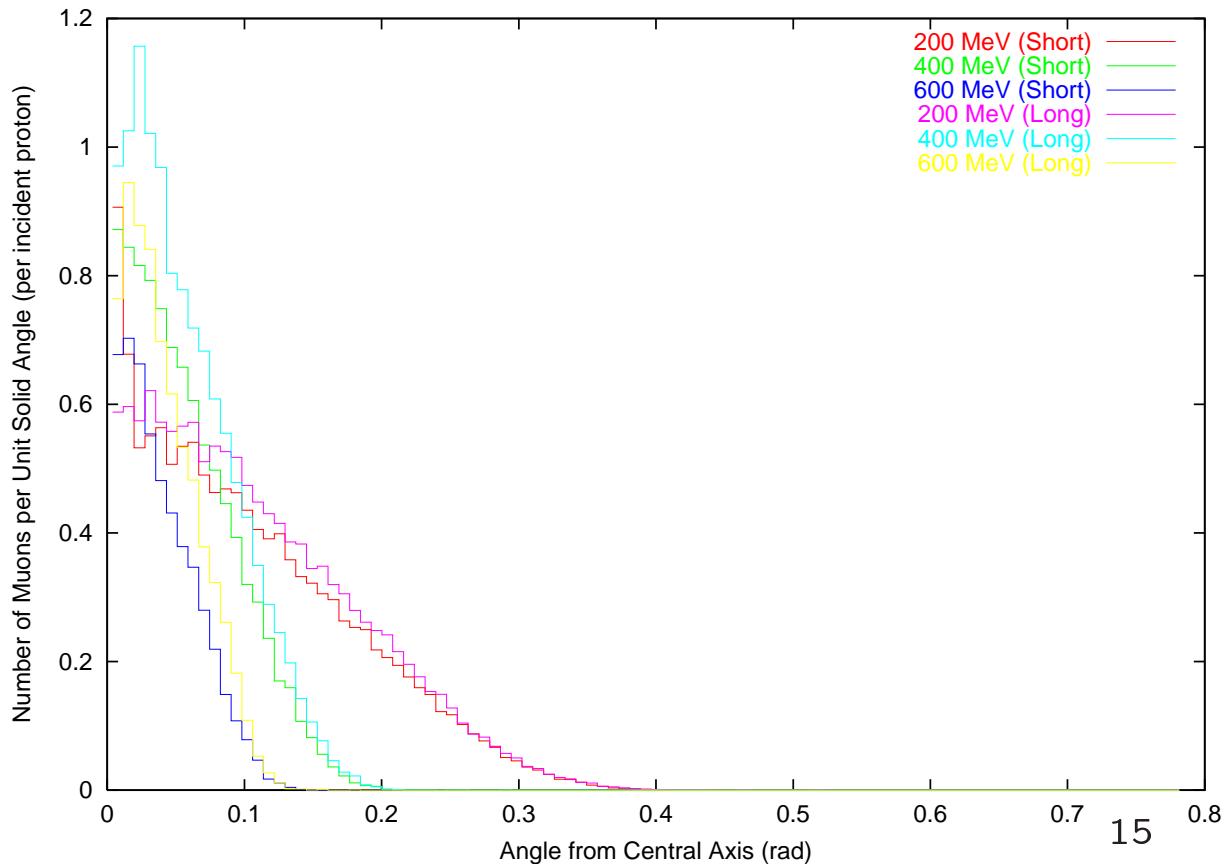
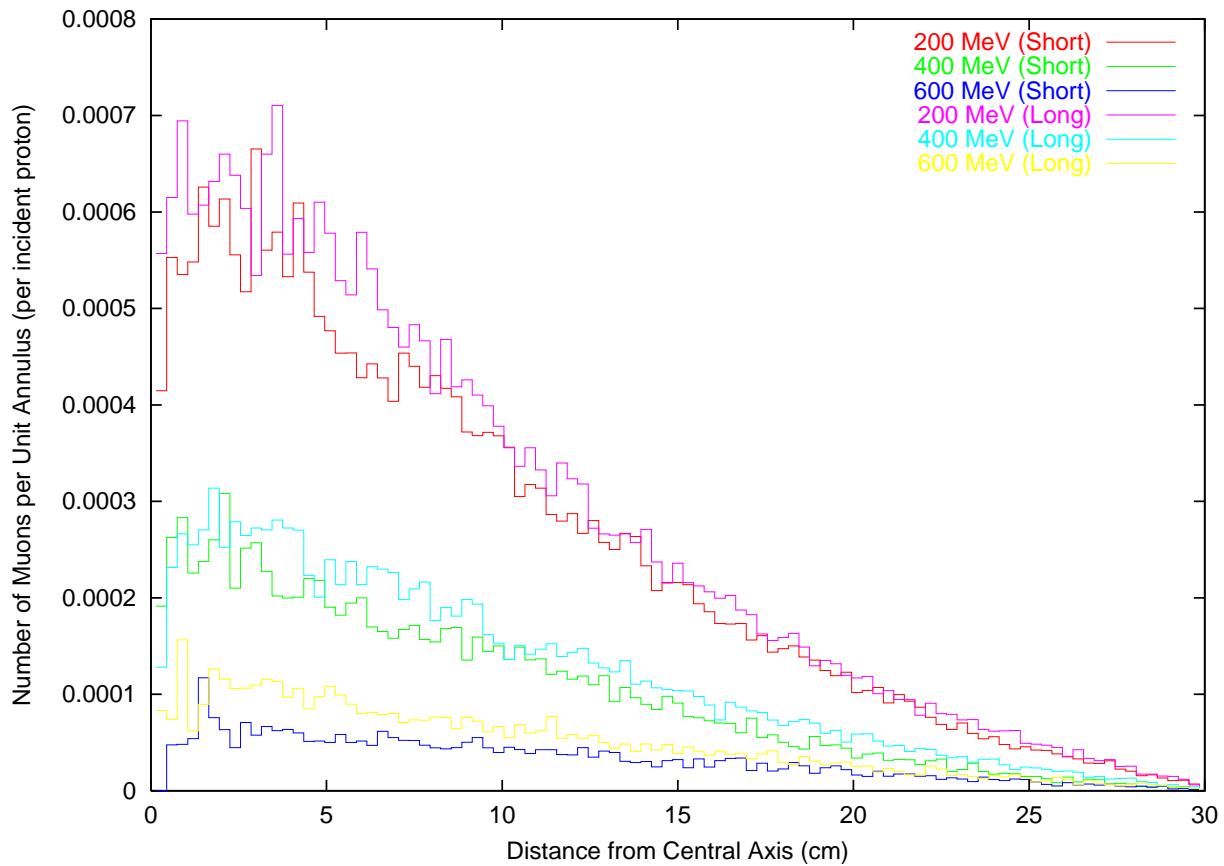
Comparable with FS2 yields with Hg jet

...Final momentum distributions (*unscaled*)



“Adiabaticity” is a big deal!

...Final phase space distributions



- Further improvements...

...In the decay channel:

$$(p_T)_{MAX} \sim 28 \text{ MeV}$$

...During pion decay:

$$\begin{aligned} \langle p_T \rangle &\sim \frac{1}{2} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \langle \sin^2 \theta \rangle \\ &= \frac{1}{4} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \\ &\approx 15 \text{ MeV} \end{aligned}$$

...Decrease losses caused by “smearing”
from pion decay by increasing $(p_T)_{MAX}$!

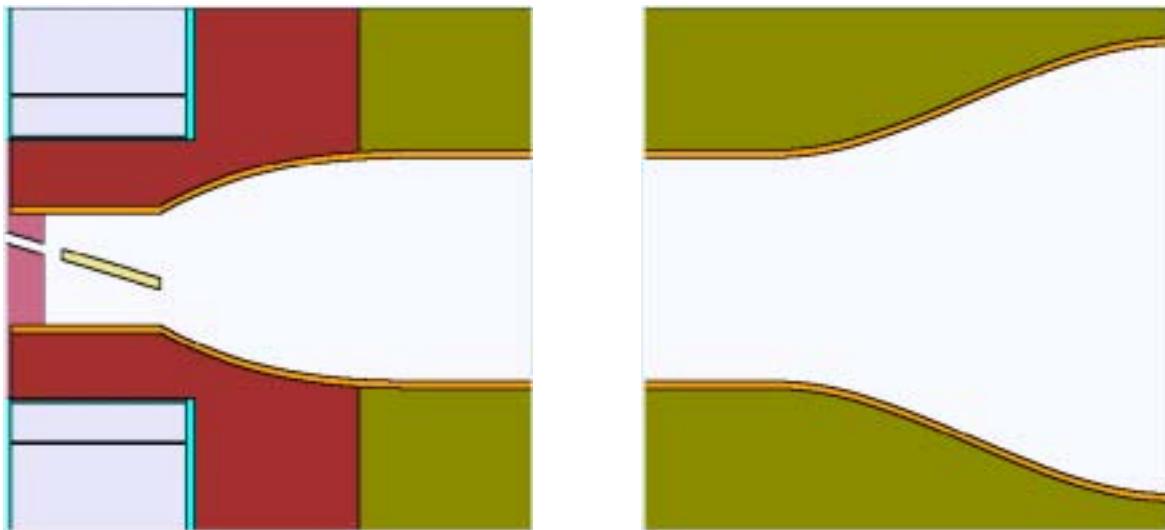
...In a 5 T decay channel:

$$(p_T)_{MAX} \sim 56 \text{ MeV}$$

with a 30 cm ($R = 15$ cm) aperture

- New horns...

...Before and after the 5 T decay channel:

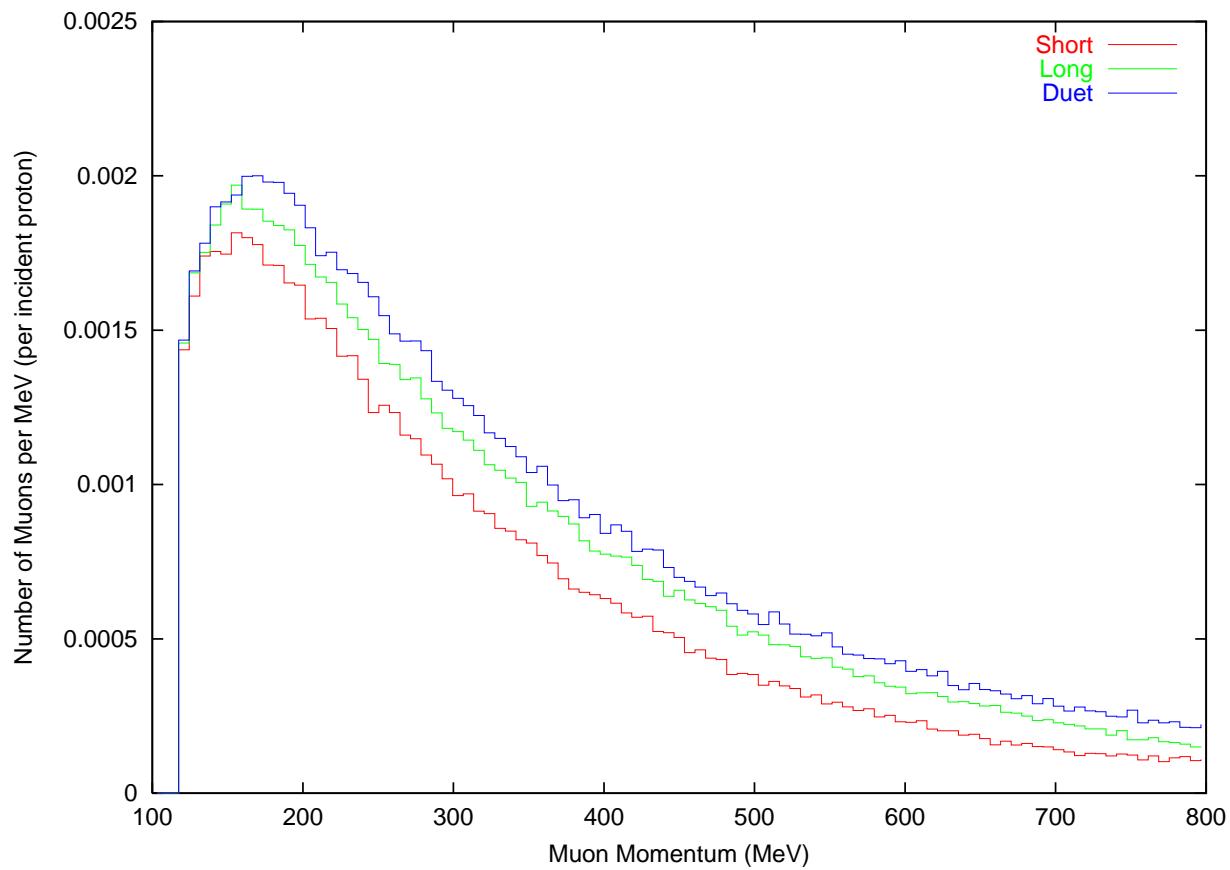


(“ehem...Two Horns is a Duet!”)

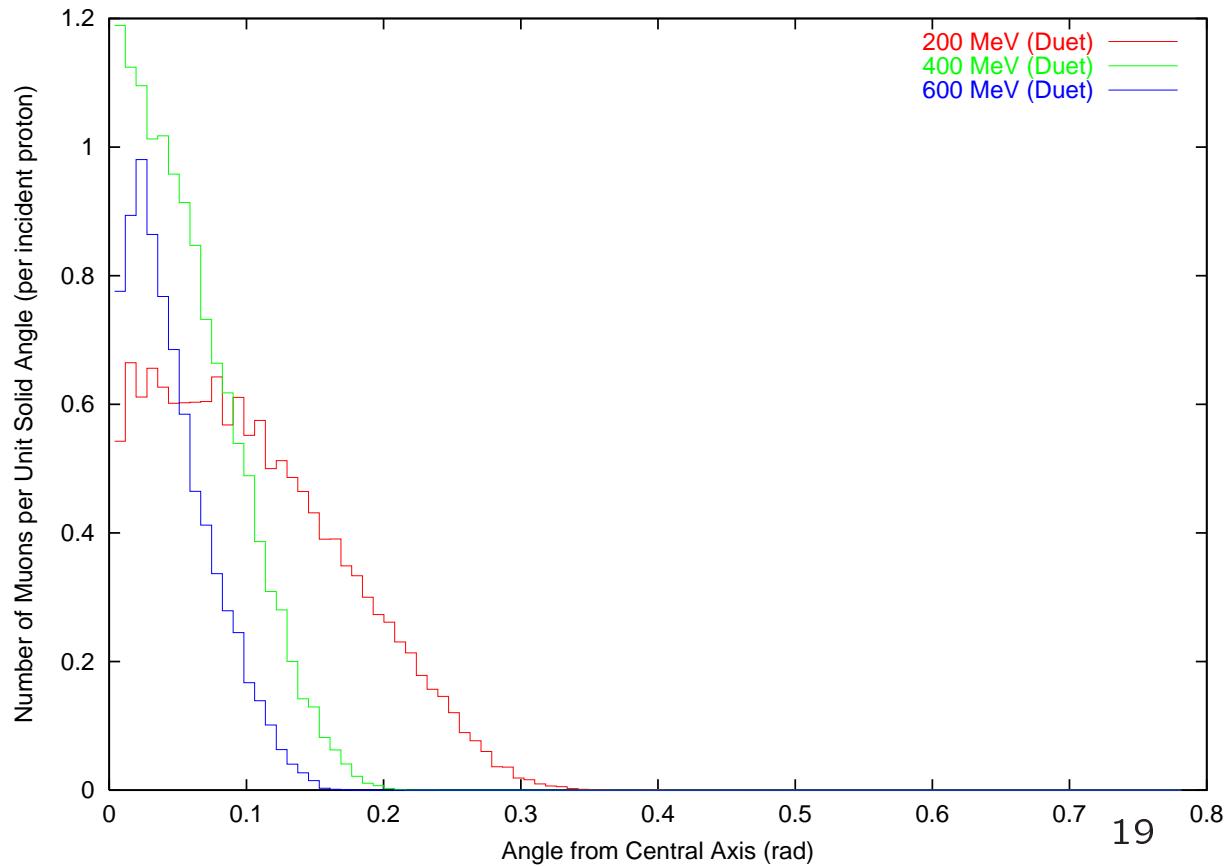
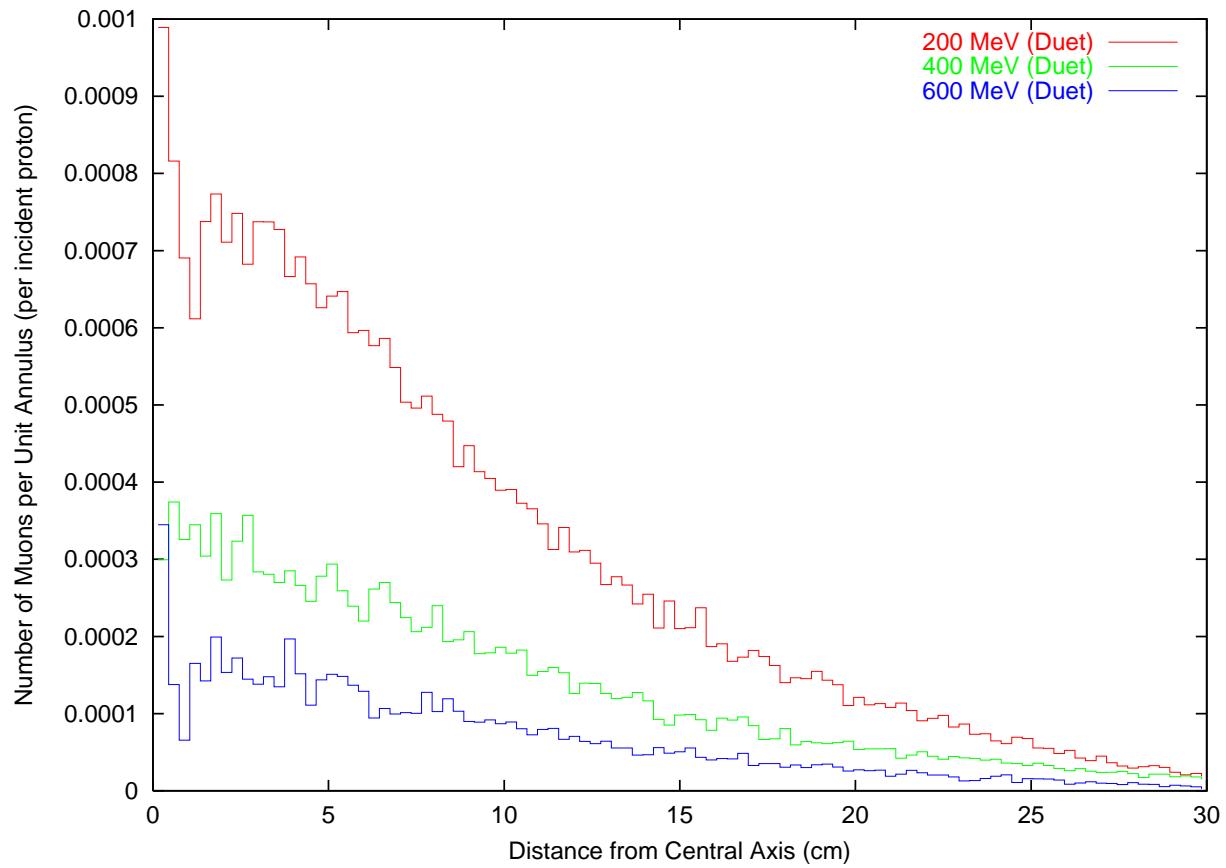
...Calculate muon yields:

DUET: $0.315(+), 0.285(-) \mu/P$

Duet momentum distribution:



Duet phase space distributions



- Things left undone...

- ...No energy deposition calculations

- ...Currently using Maxwellian but
“unrealistic” magnetic fields

- ...Possibly shorter decay channel (cheaper)

- ...“Orange”-type capture magnet
(Only captures one sign)